that the reviewer feels the book will be more useful to applied statisticians than to the author's intended audience, i.e., research workers in the experimental sciences.

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49[L].—G. F. Miller, *Tables of Generalized Exponential Integrals*, National Physical Laboratory Mathematical Tables, Vol. 3, British Information Services, New York, 1960, iii + 43 p., 28 cm. Price $1.43 postpaid.

According to the author, the tables under review were prepared to meet the requirements of quantum chemists concerned with the evaluation of molecular integrals, who frequently have found the tables computed by the New York Mathematical Tables Project and edited by G. Placzek [1] inadequate for this purpose.

Actually tabulated in the present work is the auxiliary function

$$F_n(x) = (x + n)e^x E_n(x),$$

where $E_n(x)$ represents the generalized exponential integral, defined by the equation $E_n(x) = \int_1^\infty e^{-xt}t^{-n}\, dt$. Three tables of $F_n(x)$ to 8D are provided. The first table covers the range $x = 0(0.01)1$ for $n = 1(1)8$; the second, the range $x = 0(0.1)20$ for $n = 1(1)24$; and the last, the range $1/x = 0(0.001)0.05$ for $n = 1(1)24$. Modified second (and occasionally fourth) central differences are provided throughout for use with Everett's interpolation formula. For details of methods of interpolation and tables of interpolation coefficients the table-user is referred to the first two volumes of this series of tables [2], [3].

It is stated that the total error in an unrounded interpolated value of $F_n(x)$ derived from the present tables need never exceed $1\frac{1}{2}$ units in the eighth decimal place if the tabulated differences are used. Furthermore, the values of $F_n(x)$ here tabulated are guaranteed to be accurate to within 0.6 unit in the last place.

The tables are preceded by an Introduction containing a brief account of pertinent literature, followed by a section devoted to a description of the tables and a justification for the tabulation of $F_n(x)$ in preference to $E_n(x)$. The properties of the generalized exponential integral, many of them reproduced from Placzek [1], are enumerated in a third section. The fourth section of the text is devoted to a careful description of the several procedures followed in the preparation of the tables. An excellent set of references is appended to this introductory textual material.

The typography is uniformly excellent, and the format of the tables is conducive to their easy use. The only defect observed was a systematic error in the heading of Table 2 on pages 24 through 37, where this heading erroneously appears as Table 3.

J. W. W.

1. National Research Council of Canada, Division of Atomic Energy, Report MT-1, *The Functions* $E_n(x) = \int_1^\infty e^{-xu}u^{-n}\, du$, Chalk River, Ontario, December 1946. Reproduced in

Nat. Bur. Standards Appl. Math. Ser. No. 37, *Tables of Functions and of Zeros of Functions*, 1954, p. 57–111. See RMT **392**, *MTAC*, v. 2, 1946–47, p. 272; and RMT **104**, *MTAC*, v. 10, 1956, p. 249–250.
2. L. Fox, *The Use and Construction of Mathematical Tables*, National Physical Laboratory Mathematical Tables, v. 1, London, 1956. See RMT **8**, *MTAC*, v. 13, 1959, p. 61–64.
3. L. Fox, *Tables of Everett Interpolation Coefficients*, National Physical Laboratory Mathematical Tables, v. 2, London, 1958.

50[L].—F. W. J. Olver, Editor, *Bessel Functions*, Part III, *Zeros and Associated Values*, Royal Society Mathematical Tables No. 7, Cambridge University Press, New York, 1960, lx + 79 p., 29 cm. Price $9.50.

The present volume is a step towards the completion of a program for the tabulation of Bessel functions initiated by the British Association Mathematical Tables Committee, and continued since 1948 by the Royal Society Mathematical Tables Committee. Part I of this series, *Bessel Functions, Functions of Order Zero and Unity* appeared in 1937, and Part II, *Bessel Functions, Functions of Positive Integer Order* appeared in 1952 (see *MTAC* v. 7, 1953, p. 97–98). Recall that Part I contains a section on the zeros of $J_n(z)$, $Y_n(z)$, $n = 0, 1$, but Part II is without a section devoted to zeros.

Part III, the present work, deals with the evaluation of zeros of the Bessel functions $J_\nu(z)$ and $Y_\nu(z)$ for general ν and z. Tables are also provided as described later in this review. A history of the project is given in the "Introduction and Acknowledgements," by C. W. Jones and F. W. J. Olver. A chapter on "Definitions, Formulae and Methods" by the above authors is a valuable compendium of techniques for the enumeration of zeros and associated functions. In particular, it is an excellent guide if zeros are required of other transcendental functions which satisfy second-order linear differential equations. Several methods of computation are outlined. For instance, the method of McMahon is useful for ν fixed and z large, while the inverse interpolation approach of Miller and Jones presupposes a tabulation of the functions themselves. Between the regions covered by these techniques is a gap which increases with increasing ν. The gap is bridged by application of Olver's important contributions on uniform asymptotic expansions of Bessel functions.

The section "Description of the Tables, Their Use and Preparation" is by the editor. A short description of the tables follows. Table I gives zeros $j_{n,s}$ of $J_n(x)$, $y_{n,s}$ of $Y_n(x)$, and the values of $J_n'(j_{n,s})$, $Y_n'(y_{n,s})$. Table II gives zeros $j'_{n,s}$ of $J_n'(x)$, $y'_{n,s}$ of $Y_n'(x)$, and the values of $J_n(j'_{n,s})$, $Y_n(y'_{n,s})$. Table III gives zeros $a'_{m,s}$, $b'_{m,s}$ of the derivatives $j_m'(x)$, $y_m'(x)$ of the spherical Bessel functions $j_m(x) = (\pi/2x)^{1/2}J_{m+1/2}(x)$, $y_m(x) = (\pi/2x)^{1/2}Y_{m+1/2}(x)$, and the values of $j_m(a'_{m,s})$, $y_m(b'_{m,s})$. The ranges covered are

$$n = 0(\tfrac{1}{2})20\tfrac{1}{2}, \quad s = 1(1)50, \quad \text{Tables I and II};$$
$$m = 0(1)20, \quad s = 1(1)50, \quad \text{Table III}.$$

All entries are to eight decimals, and in no case should the end-figure error exceed 0.55 of a unit in the eighth decimal.

The coefficients in the uniform asymptotic expansions (previously mentioned) which are used to evaluate items in Tables I–III for n (or m) large are given in Table IV. The expansions for the Bessel functions of Tables I–III also depend on zeros and associated values of certain Airy functions and their derivatives. These